

VII. *Determination of the Heliocentric Longitude of the descending Node of Saturn. By Thomas Bugge, Professor of Astronomy in the University of Copenhagen; communicated by Sir Joseph Banks, Bart. P. R. S.*

Read December 7, 1786.

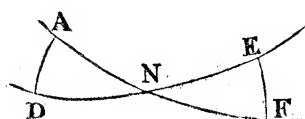
THE culmination of Saturn was observed with a 6-feet achromatic transit-instrument, and the planet compared with α and π of Sagittarius, whose apparent right-ascensions in the middle of August 1784 were $282^{\circ} 56' 54''$ and $284^{\circ} 14' 33''$. The meridian altitude was observed with a 6-feet mural quadrant. The original observations are to be published in the second volume of my Astronomical Observations. From those are calculated the right-ascension and declination, the geocentric longitude and latitude, of Saturn, which are to be depended upon to 4 or 6 seconds. Those observed longitudes and latitudes are compared with the tables of Dr. HALLEY and of M. DE LA LANDE. In the errors of the tables + signifies that the longitude of the tables is less than the observed longitude; and the meaning of - is, that the calculated longitude is greater than the observed. It ought to be observed, that the heliocentric longitudes of Dr. HALLEY's Tables have been corrected for the perturbations after the principles of M. LAMBERT. (Memoires de Berlin, pour 1783, p. 216. and Collection des Tables Astronomiques de Berlin, tom. II. p. 269.)

1784	$\frac{1}{2}$ culmi- nation, mean time at Copen.			$\frac{1}{2}$ observed longitude.			$\frac{1}{2}$ observ- ed lati- tude.			The error of HALLEY		The error of M. DE LA LANDE.				
										in long.	in lat.	in long.	in lat.			
	h.	'	" ^{s.}	°	'	" ^o	°	'	" ^B	'	"	'	"			
July 12	12	3	11	19	20	34	48	0	3	35	+ 2	22	+ 32	- 9	40	+ 33
20	11	29	9	9	19	59	39	0	2	59	+ 2	14	+ 38			
Aug. 1	10	38	25	9	19	9	22	0	1	44	+ 1	45	+ 27	- 9	49	+ 28
8	10	9	0	9	18	42	56	0	1	2	+ 1	48	+ 21			
21	9	14	59	9	18	1	23	0	0	2	+ 1	35	+ 28			
27	8	50	19	9	17	46	19	0	0	29A	+ 1	26	- 26	- 9	35	- 30
31	8	33	47	9	17	38	7	0	0	53	+ 1	19	- 23			
Sept. 5	8	13	45	9	17	29	36	0	1	26	+ 1	20	- 25	- 9	25	- 27
15	7	33	45	9	17	19	39	0	2	4	+ 1	18	- 25			
Oct. 8	6	4	23	9	17	34	6	0	3	57	+ 1	22	- 26	- 8	48	- 32

In order to reduce the observed geocentric longitude to the sun, or by observation to find the heliocentric longitude, it is required to know the angle at the planet $= p$. If this angle is calculated in the common way only by the tables, there will arise some difference, according to the different elements and the different constructions of the tables. Thus, at the time of Saturn's culmination, this angle is found the 12th of July, by the tables of Dr. HALLEY $= 0^{\circ} 3' 13''$, and by the tables of M. DE LA LANDE $= 0^{\circ} 2' 0''$; the 8th of August by HALLEY $= 2^{\circ} 43' 35''$, and by M. DE LA LANDE $= 2^{\circ} 42' 34''$; the 27th of August after Dr. HALLEY $= 4^{\circ} 14' 15''$, and after M. DE LA LANDE $= 4^{\circ} 13' 47''$. To avoid those differences, which often may alter the heliocentric longitude more than one or two minutes, the following method may be useful. The heliocentric longitude of the earth, calculated after the tables of M. MAYER, is to be depended upon to eight or ten seconds. From the heliocentric longitude of the earth, and from the observed geocentric longitude of the planet, corrected for the aberration and nutation, is deduced the angle at the earth $= i$,

or the distance between the sun and the planet seen from the earth. The dimensions of the elliptical orbit of the planet are so far ascertained, that the logarithms of the distance from the sun have not any material difference in the different tables. From the angle t , the distance of the earth from the sun, and the distance of the planet from the sun, the angle p is calculated to a sufficient degree of accuracy. Thus, the 12th of July, by the distances of Dr. HALLEY, $p = 0^\circ 2' 59''$, and by the distances of M. DE LA LANDE $p = 0^\circ 2' 59''$; the 8th of August after Dr. HALLEY $p = 2^\circ 43' 25''$, and after M. DE LA LANDE $p = 2^\circ 43' 36''$; the 27th of August after Dr. HALLEY $p = 4^\circ 14' 10''$, and after M. DE LA LANDE $p = 4^\circ 14' 28''$. The difference very seldom will amount to 20 seconds, and is of no consequence in this matter. From the observed geocentric latitude of the angle at the sun $= s$, and the angle at the earth $= t$, the heliocentric latitude of the planet is found = $\frac{\text{tang. lat. geoc.} \times \sin. s}{\sin. t}$.

1784	Mean time at Copenhagen.	$\frac{1}{2}$ observed heliocentric longitude.	$\frac{1}{2}$ observed heliocentric latitude.
	h. ' "	s. ° ' "	° ' " B
July 12	12 3 1	9 20 37 29	0 3 13 B
20	11 29 9	9 20 51 53	0 2 41
Aug. 1	10 38 85	9 21 13 17	0 1 34
8	10 9 0	9 21 26 2	0 0 56
21	9 14 59	9 21 49 27	0 0 2
27	8 50 19	9 22 0 12	0 0 27 A
31	8 33 47	9 22 7 32	0 0 50
Sept. 5	8 13 45	9 22 16 28	0 1 21
15	7 33 45	9 22 34 32	0 1 59
Oct. 8	6 4 23	9 23 16 15	0 3 35



When two heliocentric longitudes, and the corresponding northern and southern latitude are given, the distance of

of the node from one of the longitudes or places may be found. Let DE be the ecliptic, AF the orbit of the planet, N the node, DE the difference between the two observed heliocentric longitudes $= a$, EF the southern latitude $= \beta$, AD the northern latitude $= b$, NE the distance of the node from the heliocentric place at E, and corresponding to the southern latitude $= x$. In the spherical triangles ADN and FEN,

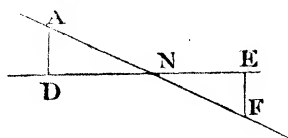
$$\frac{\sin. (a-x)}{\text{tang. } b} = \cot. N = \frac{\sin. x}{\text{tang. } \beta}.$$

By placing the value of $\sin. (a-x)$ in the equation $\frac{\sin. a \cdot \cos. x - \sin. x \cdot \cos. a}{\text{tang. } b} = \frac{\sin. x}{\text{tang. } \beta}$. By resolving

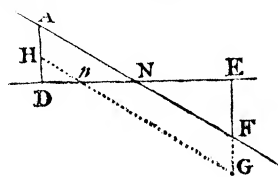
$$\text{this equation } \frac{\sin. x}{\cos. x} = \text{tang. } x = \frac{\sin. a \cdot \text{tang. } \beta}{\text{tang. } b + \cos. a \cdot \text{tang. } \beta}.$$

If a , b , and β , are very small arcs, which commonly is the case with the planets, then $\sin. a = a$, $\text{tang. } \beta = \beta$, $\text{tang. } b = b$, and $\cos. a = 1$. Hence the spherical formula will be transformed into another $x = \frac{a\beta}{b+\beta}$. This formula belongs to

plane geometry, and may besides be thus demonstrated. DN : NE = AD : EF. Hence DN + NE : NE = AD + EF : EF; and NE = $\frac{DE \times EF}{AD + EF}$. If



the difference of the longitudes do not exceed one degree, and the latitudes are not greater than ten minutes, the spherical and the rectilinear formula will agree to very few seconds. Small faults in the longitude will not very much alter the true place of the node; but very small errors in the latitude are of great consequence. Let the error in the

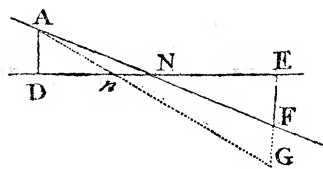


southern heliocentric latitude be $FG = +d$. The error in the northern latitude $AH = -d$. Hence $DH : Dn = GE : En$, and $En = \frac{a(\beta+d)}{b+\beta}$. By subtracting EN

=

$\frac{a\beta}{b+\beta}$, the error in the heliocentric longitude of the node,

$Nn = \frac{ad}{b+\beta}$. If the fault in the southern latitude $= -d$, in the northern latitude $= +d$, the same formula is still true; but then $EN > En$, and the place of the erroneous node will be between E and N. In both cases the errors in the place of the node are directly as the errors in the latitudes.



Let us now suppose, that only the one latitude is erroneous $\pm d$. Then

$$Nn = \pm En \mp EN = a \times \left(\pm \frac{\beta \pm d}{b + \beta \pm d} \mp \frac{\beta}{b + \beta} \right) = \frac{abd}{(b + \beta)^2 \pm d(b + \beta)}.$$

In the case when the error in both latitudes is positive $= +d$, and $\beta > b$, or $\beta < b$, the resulting error in the place of the node $=$

$$\frac{ad(\mp b \pm \beta)}{(b + \beta)^2 + 2d(b + \beta)}.$$

In the case when the error in both latitudes is negative $-d$, and $\beta > b$, or $\beta < b$, then the error in the node $=$

$$\frac{ad(\mp b \pm \beta)}{(b + \beta)^2 - 2d(b + \beta)}.$$

In those two cases the error is less than in any of the former, and quite nothing when $b = \beta$. If the radius of the instrument, with which the meridian altitudes are observed, is given, the quantity of d is also given. In a mural quadrant of 6 or 8 feet $d = 5$ or 3 seconds. Take $a = 34' 3''$, $b = 56''$, $\beta = 27''$, $d = 5''$, and the error in the southern latitude $+d$, in the northern $= -d$; then $Nn = \frac{10215''}{83} =$

$2' 12''$. Take now the error only in the southern latitude

$$= +d; \text{ then } Nn = \frac{572040''}{7304} = 1' 18''; \text{ in the case of } -d; Nn =$$

$$\frac{572040''}{6474} = 1' 28''. \text{ From hence it appears, that in comparing}$$

two single observations, it scarce will be possible to avoid a fault of ± 2 minutes in the place of the node.

If the instrument is of a less force than a mural quadrant of 6 feet, and the possible faults in the altitudes greater, for example, 10 or 15 seconds, the resulting error in the place of the node may very easily be calculated; but the error in the node will be enormous, and the observations of no use for a nice astronomer.

Compared Observations 1784.	η heliocentric longitude on the last day.	η distance from the $\odot$.	Heliocentric longitude of η $\odot$
	s. ° ' "	° ' "	s. ° ' "
July 12 with Sept. 15	9 22 34 32	0 44 38	9 21 49 54
July 12 — Oct. 8	9 23 16 15	1 23 41	9 21 52 36
July 20 — Sept. 15	9 22 34 32	0 43 37	9 21 50 55
July 20 — Oct. 8	9 23 16 15	1 22 33	9 21 53 42
Aug. 1 — Sept. 5	9 22 16 28	0 29 15	9 21 47 13
Aug. 1 — Sept. 15	9 22 34 32	0 45 23	9 21 49 9
Aug. 1 — Oct. 8	9 23 16 15	1 25 34	9 21 50 41
Aug. 8 — Aug. 27	9 22 0 12	0 11 7	9 21 49 5
Aug. 8 — Aug. 31	9 22 7 32	0 19 34	9 21 47 58
Aug. 21 — Aug. 27	9 22 0 12	0 10 0	9 21 50 12
Aug. 21 — Aug. 31	9 22 7 32	0 17 23	9 21 50 9
Mean			9 21 50 8,5

This mean agrees pretty well with the observations on the 21st, 27th, and 31st of August, which are nearest the node, and most to be depended upon.

The 21st of August, at 9 h. 12' 26" true time at Copenhagen, the heliocentric longitude of Saturn = 9 s. 21° 49' 27", and the distance from the node = 41". The 27th of August, at 8 h. 49' 23", the heliocentric longitude = 9 s. 22° 0' 12"; therefore, in 5 days 23 h. 36' 57" Saturn has described an arc of 10' 45", and $10' 45'' : 5 \text{ d. } 23 \text{ h. } 36' 57'' = 41'' : x$. Hence

Saturn has spent 9 h. 7' 44'' in going through those 41''; and *Saturn's passage through the node happened August 21, 1784, at 18 h. 20' 10'', and the heliocentric longitude of his descending node = 9 s. 21° 50' 8'',5.* The errors in the place of the node are relative to the tables of Dr. HALLEY + 19' 39'', to the tables of M. CASINI + 16' 4'', and to the tables of M. DE LA LANDE + 1' 31''.

In the foregoing computation of Saturn's heliocentric longitude from the tables of Dr. HALLEY, this longitude has been corrected for the perturbation after the principles of M. LAMBERT. Though the geocentric places, calculated in this manner, will agree still better with the observations than without those perturbations, nevertheless they are only empiric, and not founded upon the theory and principles of gravitation; I shall therefore conclude this Paper, by adding the faults in the heliocentric places of Saturn, calculated only and directly from the tables of Dr. HALLEY, which may be of some use to improve those valuable tables,

1784	h heliocentric longitude from Dr. HALLEY's tables.	Error in longitude.	h heliocentric latitude from Dr. HALLEY's tables.	Error in latitude.
	s. ° ' "	' "	° ' " B	+ "
July 12	9 20 27 40	+ 9 49	0 2 45 B	+ 28
20	9 20 42 3	+ 9 50	0 2 17	+ 24
Aug. 1	9 21 3 41	+ 9 36	0 1 10	+ 24
8	9 21 16 19	+ 9 43	0 0 37	+ 21
21	9 21 39 45	+ 9 42	0 0 24 A	- 22
27	9 21 50 34	+ 9 38	0 0 53	- 26
31	9 21 57 47	+ 9 45	0 1 11	- 21
Sept. 5	9 22 6 48	+ 9 40	0 1 45	- 24
15	9 22 24 50	+ 9 42	0 2 22	- 23
Oct. 8	9 23 6 33	+ 9 44	0 4 1	- 26

